

## Effects of Hydraulic Residence Time and Mixing on Wastewater Treatment in a Membrane Bioreactor

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*Dedicated to Prof. Egon Bauman*

Effects of hydraulic residence time distribution (HRTD) and degree of mixing on aerobic wastewater treatment in a membrane bioreactor are studied. Experimentally are determined HRTD and the life expectancy functions  $\Lambda$  in a 40 L Zenon (ZW 10) hollow-fiber submerged microfiltration membrane bioreactor at liquid flow rate range of 7.4 – 36 L h<sup>-1</sup> and gas flow rate range 1.7 – 5.1 m<sup>3</sup> h<sup>-1</sup>. The HRTD functions are modelled by a series of two parallelly connected CSTR corresponding to slow and fast liquid flow. The model approximates the HRTD functions with an average relative error of 5 %. A model for prediction of average liquid residence time based on superficial liquid and gas velocities is developed. From HRT functions are evaluated the life expectancy functions  $\Lambda$  of dissolved biodegradable substrates and oxygen in liquid phase. Parameters from Activated Sludge Model No. 1 (ASM1) for aerobic degradation by heterotrophic biomass are applied for computer simulation. Concentration of soluble biodegradable substances in the outlet stream is the measure of effects of HRT and mixing. Applied are the following models: ideal mixing, segregated flow, maximum mixedness, and the CSTR network. With reference to the network model, determined are relative differences of outlet concentration of biodegradable substrate 59 %, 86 % and 32 % corresponding to the flow models relative to the CSTR network model.

*Keywords:*

HRT, membrane bioreactor, degree of mixing, wastewater treatment

### Introduction

Aerobic membrane reactors with suspended active sludge of a microbial mixed culture are among the most complex bioreactors to analyse from the mathematical point of view. The complexity is due to lack of detailed experimental data of microbial activity of a mixed culture with present suspended waste components, and intricate interaction of hydrodynamic shear rate between gas bubbles, liquid and microorganism metabolism. Mathematical models of bioreaction kinetics involve multisubstrate limitations and inhibitions, and continuous dynamical changes of mixed culture composition due to adaptation to continuously varying, and often with undetermined, substrate composition, such as in the case of municipal wastewater. Besides the intractability of detailed microbial kinetics, just the problem of mathematical modelling of complex hydrodynamic nonstationary multiphase behaviour is a formidable task. It requires solution of Navier-Stokes equation for gas and liquid with supplied rheological and/or turbulence models coupled with reac-

tion rates. A numerical solution is attainable by CFD – computational fluid dynamic methods that are based on extensive space and time meshing and use of finite element approximation of solution of Navier-Stokes equation.<sup>1</sup> From a simplified reactor engineering view, hydrodynamic effects in a multiphase reactor are usually described on macroscopic scale by probability residence time density function (RTD), named for liquid hydraulic residence time distribution (HRTD), and gas residence time distribution (GRTD).<sup>2–4</sup> However, the hydrodynamic effects on microscopic scale and knowledge of RTD function does not suffice for a determined evaluation of effects on chemical and biochemical reactions, and on a total bioreactor performance.<sup>4–5</sup> Orders of biochemical reactions vary in the range from –1 to +1 depending on substrate concentration leading to difficulties in prediction of effect of degree of mixing on bioconversions.<sup>6–8</sup>

Recently membrane bioreactor (MBR) has attracted considerable attention for wastewater treatment. It provides numerous advantages, essential integration of biological activity and biomass retention in a single working volume, very high biomass

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retention time needed for development of mixed culture with simultaneous nitrification and denitrification activity, and effective phosphorus removal.<sup>10–15</sup> However, introduction of submerged membrane in a bioreactor requires intensive fluid and air bubble movement at membrane surface. The main objective is to maintain low membrane hydraulic resistance (high porosity) by action of liquid and air bubble shear forces at a membrane surface. Also, the strong shear forces break up microbial flocs, reduce aggregates of waste particles, and enables breakage of suspended filamentous networks.

In view of the importance of hydrodynamics effects, the aim of this work is to study experimentally residence time distribution and to determine effects of degree of mixing on the biodegradation reaction of waste by use of computer simulation based on Activated Sludge Model No. 1 (ASM1).<sup>16–17</sup> Applied are numerical methods based on mass and reaction balances for the boundary cases of the earliest and the latest micromixing. The boundary cases of micromixing correspond to the models of maximum mixidness and segregated flow respectively. Furthermore, on macroscopic scale determined is a network of ideally mixed tanks as a model for numerical evaluation of HRTD functions.

## Materials and methods

A laboratory scale study of hydraulic retention time distribution (HRTD) was conducted in 55 L Zenon (ZW 10) hollow-fiber submerged micro-filtration membrane bioreactor (Fig. 1). The ZW-10 module, length 692 mm and diameter 109 mm, is placed vertically and comes with an extended aera-



Fig. 1 – Top view of Zenon (ZW 10) hollow-fiber micro-filtration membrane bioreactor

tion tube. The permeate is drawn from the top header, and the centre aeration tube supplies air to the bottom header where air diffusers are located. To investigate dynamics of mixing and retention distribution experiments were conducted at liquid flow rate range of 7.4 – 36 L h<sup>-1</sup> and gas flow rate range 1.7 – 5.1 m<sup>3</sup> h<sup>-1</sup> at the reactor working volume of 40 L. Dimensions of the membrane were 692.15 mm x 109.54 mm, with total surface area 0.92 m<sup>2</sup>, and with pore size of 0.4 μm. The experimental setup is presented in Fig. 2. The reactor was continuously fed with demineralised water from the bottom, and air was continuously introduced at the point below the membrane. Liquid is continuously drawn from the membrane at the top of the reactor. A concentrated solution of NaCl was applied as a tracer. Volume of 50 mL of saturated concentration  $w_{\text{NaCl}} = 28\%$  as a short impulse (approximated by Dirac  $\delta$  function) was injected by a needle from the top of the reactor at three different positions (Fig. 2). In the experiments for measurement of mixing time the inlet and outlet tubes were connected providing a closed loop reactor. Concentration of the tracer was measured in the outlet tube from the reactor by a flow-through conductivity cell (Oakton Con 200) with RS-232 interface connected to a PC for data logging. Data were sampled at average frequency 1 min<sup>-1</sup>, and about 1000 data points are collected in each experiment. Performed were experiments at the combinations of liquid flow rates  $q_L$  (7.2; 11.25; 16.51; 36 L h<sup>-1</sup>), and gas flow rates  $q_G$  (1.7; 3.4; 5.1 m<sup>3</sup> h<sup>-1</sup>), and for each combination tracer was injection at the three points, denoted as A, B, and C in Fig. 2.

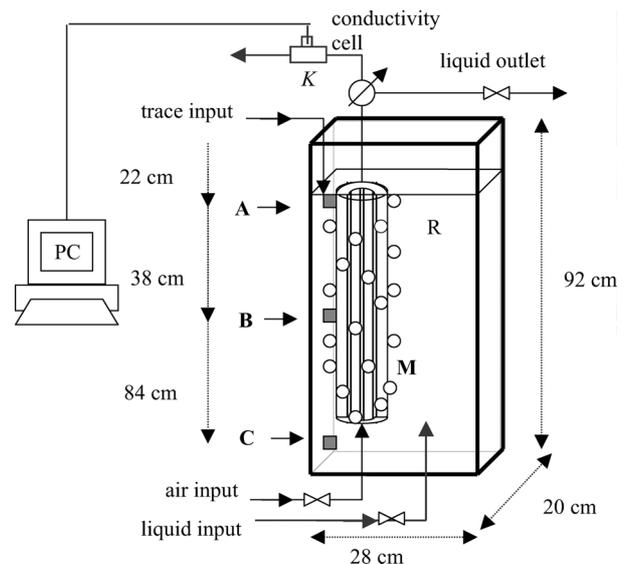


Fig. 2 – Schematic diagram of the experimental set-up for determination of mixing time and hydraulic residence time distribution, HRTD. (R) reactor; (M) membrane; (K) flow-through conductivity cell, feeding points of the tracer (A,B,C) are marked as (□), and air bubbles are depicted by circles (○).

For data analysis and model simulation for biodegradable degradation software system *Mathematica* is applied.<sup>18</sup>

## Mathematical model

In closed loop tank experiments responses of the tracer enable determination of the mixing time. In Fig. 3 are presented typical responses at different liquid and gas flow rates showing that average mixing time is about 8 minutes irrespective of the gas and liquid flow rates in the given range of experimentation.

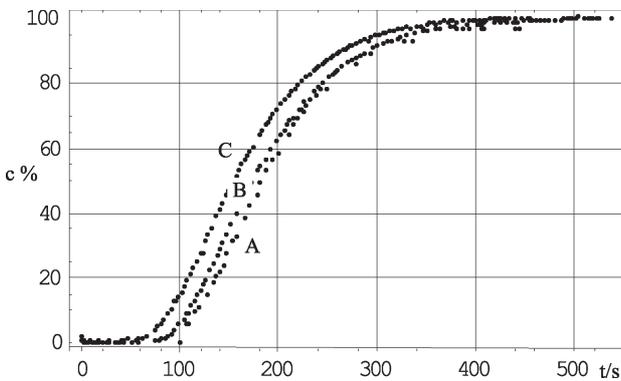


Fig. 3 – Responses of concentration in the mixing experiments at the following liquid and air flowrates: A)  $q_L = 7.2$  L/h and  $q_G = 1.7$  m<sup>3</sup>/h; B)  $q_L = 7.2$  and  $q_G = 5.1$  m<sup>3</sup>/h; C)  $q_L = 11.25$  and  $q_G = 1.7$  m<sup>3</sup>/h

From the tracer concentration responses in steady flow through experiments the average residence time  $\bar{\tau}$  numerically are evaluated by integration of:

$$\bar{\tau} = \int_0^{\infty} t \cdot E(t) \cdot dt \quad (1)$$

where  $E(t)$  is the density of residence time distribution function. The density function was obtained by the first order numerical interpolation of the tracer concentration data and subsequent area normalisation to unity. The density distribution of the life expectancy function  $\Lambda(t)$  is determined from the residence density function  $E(t)$  by:

$$\Lambda(t) = \frac{E(t)}{1 - \int_0^t E(t) \cdot dt} \quad (2)$$

An assumed network of continuous stirred tank is applied to model the experimentally determined distributions. A simple network structure is given in Fig. 4. It is composed of two parallel tanks serially

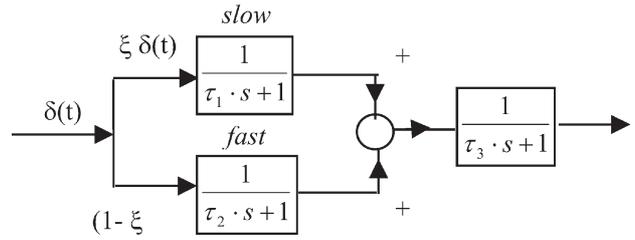


Fig. 4 – Model of hydraulic residence time distribution (HRTD) with depicted “slow”, “fast”, and “mixing” zones

connected to the third tank. By the parallel connections of tanks are modelled two streams of liquid with significantly different velocities. The slow stream is related to the speed of liquid pumping, while the fast stream is a result of drag force of air bubbles on liquid, and is mostly determined by the bubble raise velocity and gas flow rate. The two streams mix into the membrane fibers which in the model corresponds to the third parallelly connected tank. The inert mass balance in Laplace domain provides mathematical evaluation of flow dynamics and mixing. Dynamics of each tank is modelled as a first order system, i.e. with a transfer function with a single real and negative pole  $\tau$ . The gains of the parallel tanks correspond to the stream flow distribution coefficient  $\varepsilon$ . The network transfer function is given by:

$$w(s) = \left( \varepsilon \frac{1}{\tau_1 \cdot s + 1} + (1 - \varepsilon) \cdot \frac{1}{\tau_2 \cdot s + 1} \right) \cdot \frac{1}{\tau_3 \cdot s + 1} \quad (3)$$

The inverse Laplace transform of the model (3) corresponds to the density residence function of liquid:

$$HRTD(t) = E(t) = L^{-1}[W(s)] \quad (4)$$

In general, it yields a linear combination of three exponentially decaying functions which can be fitted to the experimentally determined density functions. The model parameters ( $\varepsilon$ ,  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ ) are estimated by the Levenberg-Marquardt nonlinear minimisation of the variance between experimental data and the model, separately for each experiment with selected liquid and gas flow rate.

Also, a regression model average liquid residence times  $\bar{\tau}$  is developed based on the superficial liquid  $v_L$ , superficial gas velocity  $v_G$ , and the position of injection of the tracer  $h$ . The “power law” model is expressed by:

$$\bar{\tau} = b_0 \cdot v_L^{b_1} \cdot v_G^{b_2} \cdot h^{b_3} \quad (5)$$

The model parameters were estimated by the minimisation of the variance between experimen-

tally observed average residence time at given liquid and gas flow rates.

The effects of the residence time distribution on performance of the bioreactor on municipal wastewater treatment were analysed by the computer simulation. Assumed are isothermal and aerobic conditions with only easily biodegradable waste components present in the reactor, and the active sludge is mainly composed of heterotrophic microorganism. The model accounts for the balance of mass concentration of suspended waste:

$$\begin{aligned} \frac{d}{dt} \gamma_S &= \frac{1}{\tau_0} \cdot (\gamma_{S,f} - \gamma_S) - \\ &- \frac{1}{Y_{X/S}} \cdot \mu_m \cdot \frac{\gamma_S}{K_S + \gamma_S} \cdot \frac{\gamma_0}{K_0 + \gamma_0} \cdot \gamma_X \end{aligned} \quad (6)$$

Based on assumption of only endogenous consumption of oxygen, the balance of mass concentration of dissolved oxygen is given by:

$$\begin{aligned} \frac{d}{dt} \gamma_0 &= k_l a \cdot (\gamma_0^* - \gamma_0) - \\ &- \frac{1}{Y_{X/O}} \cdot \mu_m \cdot \frac{\gamma_S}{K_S + \gamma_S} \cdot \frac{\gamma_0}{K_0 + \gamma_0} \cdot \gamma_X \end{aligned} \quad (7)$$

where limitation of oxygen transfer between liquid and gas is given by the specific volumetric oxygen transfer rate  $k_l a$ .

The balance of mass concentration of the active sludge assumes simultaneous biomass growth and autolysis. Assumed is Monod's kinetics with simultaneous limitation of biomass growth on biodegradable waste components and dissolved oxygen. The balance is:

$$\frac{d}{dt} \gamma_X = \mu_m \cdot \frac{\gamma_S}{K_S + \gamma_S} \cdot \frac{\gamma_0}{K_0 + \gamma_0} \cdot \gamma_X - k_d \cdot \gamma_X \quad (8)$$

Parameters from Activated Sludge Model No. 1 (ASM1) for endogenous aerobic degradation by heterotrophic biomass are applied for computer simulation.<sup>16–17</sup>

Effects of the residence distribution on reactor performance depend on the intricate interactions at molecular level described by the degree of mixing. Assuming ideal mixing in gas phase of the membrane reactor, attainable conversions of the waste consumption reactions in liquid phase can be evaluated by determination of the margins set by the ideally segregated and maximum mixidness models.<sup>4–6</sup> The upper boundary of conversion is set by the segregated flow when outlet concentrations of waste components (usually expressed as total COD in mg O<sub>2</sub> L<sup>-1</sup>) is determined by the integration of mass balances along all uninteracting stream lines. Hence, the outlet steady state mass concentrations

of substrate present in municipal wastewater and oxygen are given by:

$$\gamma_{S,\text{out}} = \int_0^\infty \gamma_S^B(t) \cdot HRTD(t) \cdot dt \quad (9)$$

$$\gamma_{0,\text{out}} = \int_0^\infty \gamma_0^B(t) \cdot HRTD(t) \cdot dt \quad (10)$$

The integrands are the products of mass concentration time profiles in an ideally mixed batch reactor (denoted with the superscript *B*) and the hydraulic residence time distribution. The batch balances are integrated with the initial conditions equal to the inlet feed concentrations for the continuous reactor:

$$\begin{aligned} \frac{d}{dt} \gamma_S^B &= - \frac{1}{Y_{X/S}} \cdot \mu_m \cdot \frac{\gamma_S^B}{K_S + \gamma_S^B} \cdot \frac{\gamma_0^B}{K_0 + \gamma_0^B} \cdot \gamma_X \\ \gamma_S^B(0) &= \gamma_{S,f} \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d}{dt} \gamma_0^B &= - \frac{1}{Y_{X/O}} \cdot \mu_m \cdot \frac{\gamma_S^B}{K_S + \gamma_S^B} \cdot \frac{\gamma_0^B}{K_0 + \gamma_0^B} \cdot \gamma_X \\ \gamma_0^B(0) &= \gamma_0^* \end{aligned} \quad (12)$$

The opposite boundary case is defined by assumption of the earliest mixing, when the maximum mixidness is achieved. In this case the mass balances involve the life expectancy function  $\Lambda(t)$  and are evaluated by a set of nonlinear differential equations:

$$\begin{aligned} \frac{d}{d\lambda} \gamma_S &= - \frac{1}{Y_{X/S}} \cdot \mu_m \cdot \frac{\gamma_S}{K_S + \gamma_S} \cdot \frac{\gamma_0}{K_0 + \gamma_0} \cdot \gamma_X + \\ &+ \Lambda(\lambda)(\gamma_S(\lambda) - \gamma_{S,f}) \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d}{d\lambda} \gamma_0 &= - \frac{1}{Y_{X/O}} \cdot \mu_m \cdot \frac{\gamma_S}{K_S + \gamma_S} \cdot \frac{\gamma_0}{K_0 + \gamma_0} \cdot \gamma_X + \\ &+ \Lambda(\lambda)(\gamma_0(\lambda) - \gamma_0^*) \end{aligned} \quad (14)$$

The asymptotic right hand boundary condition are applied:

$$\frac{d}{d\lambda} \gamma_S(\lambda)|_{\lambda=\infty} = 0 \quad (15)$$

$$\frac{d}{d\lambda} \gamma_0(\lambda)|_{\lambda=\infty} = 0 \quad (16)$$

The solution of the steady state problem is given by the initial conditions:

$$\gamma_{S,\text{out}} = \gamma_S(\lambda=0) \quad \gamma_{0,\text{out}} = \gamma_0(\lambda=0) \quad (17)$$

Integration of eq. 13–14. has to be evaluated in the reverse direction (from the right to the left) with the initial conditions determined from the nonlinear algebraic equations:

$$0 = -\frac{1}{Y_{X/S}} \cdot \mu_m \cdot \frac{\gamma_S(\lambda = \infty)}{K_S + \gamma_S(\lambda = \infty)} \cdot \frac{\gamma_0(\lambda = \infty)}{K_0 + \gamma_0(\lambda = \infty)} \cdot \gamma_X + \Lambda_\infty \cdot (\gamma_S(\lambda = \infty) - \gamma_{S,f}) \quad (18)$$

$$0 = -\frac{1}{Y_{X/O}} \cdot \mu_m \cdot \frac{\gamma_S(\lambda = \infty)}{K_S + \gamma_S(\lambda = \infty)} \cdot \frac{\gamma_0(\lambda = \infty)}{K_0 + \gamma_S(\lambda = \infty)} \cdot \gamma_X + \Lambda_\infty \cdot (\gamma_0(\lambda = \infty) - \gamma_0^*) \quad (19)$$

### Results and discussion

Results of the trace responses conducted in the closed loop tank experiments for liquid, and with continuous aeration are presented in Fig 3. Very similar results were obtained for each combination of liquid  $q_L \in [7.2 - 36 \text{ L h}^{-1}]$  and gas flow rate  $q_G \in [1.7 - 5.1 \text{ m}^3 \text{ h}^{-1}]$ , which shows that under these conditions mixing time is constant at about 8 min.

Tracer results, equivalent to the liquid residence time distribution HRTD, from the liquid flow through experiments are given in Fig. 5–7. Here are jointly presented the experimental data of the tracer concentration together with the responses of the mathematical models defined eq. 3 (network model) and of the corresponding ideally mixed tank (CSTR). The model parameters ( $\epsilon, \tau_1, \tau_2, \tau_3$ ) are estimated for each experiment by nonlinear minimisation of the variance between experimental tracer data and the prediction by eq. 3. For each experiment the model predicted HRTD function closely approximates the observed experiment, with an average error of about 5 %. The main discrepancy between the model predictions and data can be observed in the “tails” of the functions. The model distributions approach zero asymptotically while

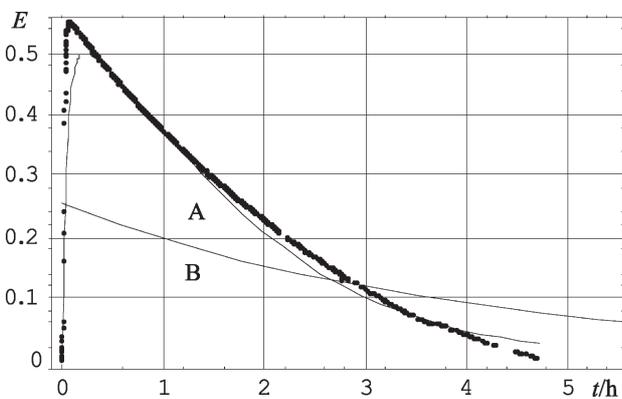
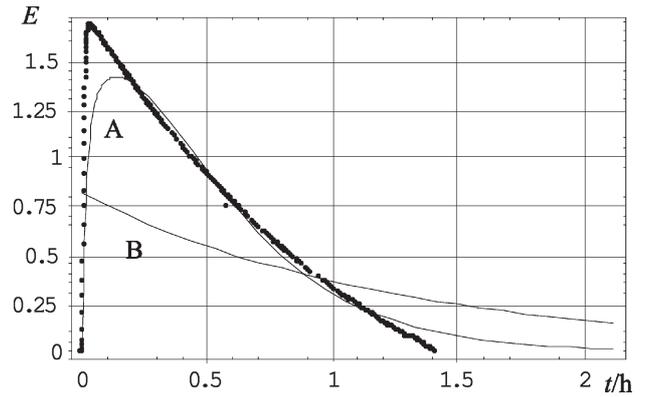
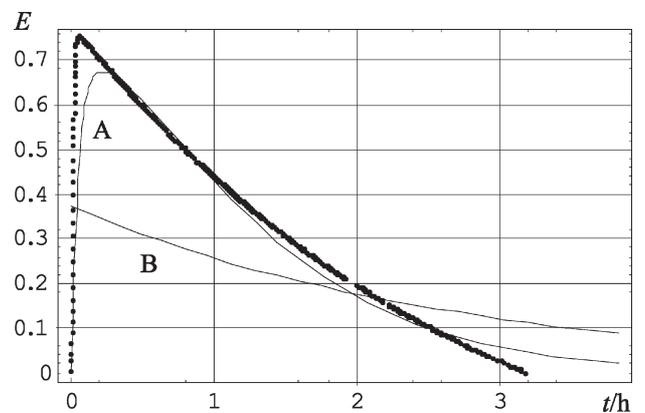


Fig. 5 – HRTD distributions at  $q_L = 11.25 \text{ L/h}$  and  $q_G = 3.4 \text{ m}^3/\text{h}$ : experimental data (••), (A) model, (B) ideal CSTR distribution



Exp. 6 – HRTD distributions at  $q_L = 36 \text{ L/h}$  and  $q_G = 5.1 \text{ m}^3/\text{h}$ : experimental data (••), (A) model, (B) ideal CSTR distribution



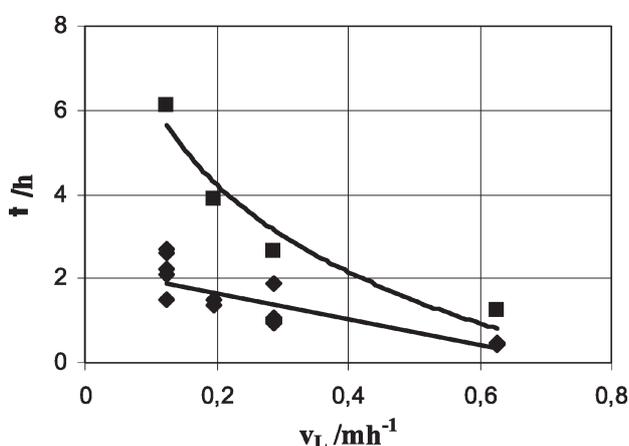
Exp. 7 – HRTD distributions at  $q_L = 16.51 \text{ L/h}$  and  $q_G = 5.1 \text{ m}^3/\text{h}$ : experimental data (••), (A) model, (B) ideal CSTR distribution

the experimental data reach zero level in a finite time. This may be partly explained by the fact that measurement cell has a finite sensitivity threshold below which measurement signal cannot be distinguished from the background signal. The estimated model parameters are highly consistent between experiments. For example, the flow distribution coefficient  $\epsilon$  is in the range 0.51 – 0.7. The highest value of the distribution coefficient is estimated for the highest liquid flow rate  $q_L = 36 \text{ L h}^{-1}$ , which can be attributed to the dominance of liquid pumping on flow in the reactor. The lowest value is estimated at the highest gas flow rate  $q_G = 3.4 \text{ m}^3 \text{ h}^{-1}$  and the minimal liquid flow rate  $q_L = 7.2 \text{ L h}^{-1}$ , when gas flow exerts the highest effect on liquid flow. The values of the time constants are in the following ranges:  $\tau_1 \in [0.3 - 1.9 \text{ h}]$ ,  $\tau_2 \in [0.015 - 0.05 \text{ h}]$ ,  $\tau_3 \in [0.3 - 1.5 \text{ h}]$ . The correlation coefficients between the model parameters and liquid and gas velocities are given in Table 1. The correlations support the arguments of the mixing model that liquid flow predominates in the “slow” and “mixing” zones (correlation  $R^2 = 0.99$ ), while in the “fast” zone there is a

Table 1 – Correlation matrix between the model parameters and liquid and gas flow rates. Significant coefficients at level  $\alpha = 0,05$  are highlighted

Superficial velocities	Slow zone	Fast zone	Mixing zone	Flow distribution coefficient.
	$1/\tau_1$	$1/\tau_2$	$1/\tau_3$	$\epsilon$
$v_L$	<b>0,99</b>	<b>0,51</b>	<b>0,99</b>	<b>0,78</b>
$v_G$	-0,05	<b>0,54</b>	-0,05	-0,31

cooperative effect of gas and liquid flow (correlation  $R^2 = 0.55$ ). Significant discrepancy between the experimentally determined residence distributions and the distribution for ideally mixed reactor is observed in Fig. 5–7. The ideal distributions have about 50 % lower density in the first hour and have predominantly extended long tails. Based on these distributions are calculated average residence times and are presented in Fig. 8. for different liquid superficial velocities. At low liquid flow rates,  $v_L < 0,2 \text{ m h}^{-1}$  observed average residence time is about 3 times lower then the predicted by assumption of ideal mixing. Difference between the two average residence values decreases by increase of liquid velocity, and is almost negligible at the highest tested liquid flow rate  $v_L \approx 0,6 \text{ m h}^{-1}$ .

Fig. 8 – Experimentally determined average hydraulic residence times ( $\blacklozenge$ ) and computed values with assumption of ideal mixing ( $\blacksquare$ ) as functions of superficial liquid velocity

In order to obtain a simple predictive model for the average residence time applied is the “power law” model given by eq. 5. The model parameters are estimated by the nonlinear Levenberg-Marquardt method yielding the following estimates:  $b_0 = 0.3647$ ,  $b_1 = -0.8953$ ,  $b_2 = -0.000143$ ,  $b_3 = 0.08276$ . The superficial velocities are given in  $v$  ( $\text{m h}^{-1}$ ), the distance  $h$  (m), and  $\bar{t}$  (h). The correlation coefficient between the observed data and model predictions is

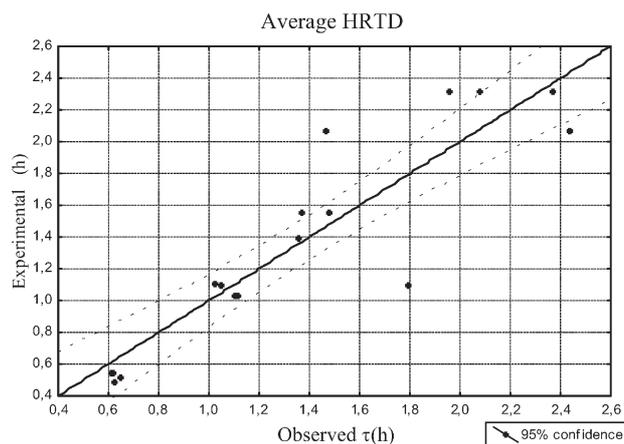


Fig. 9 – Relationship between observed and the corresponding model predictions of the average hydraulic residence time distribution HRTD at various liquid and gas flow rates at 95 % confidence

$R^2 = 0.905$ . In Fig. 9 are presented the predicted versus the observed average residence times for different combinations of gas and liquid flow rates.

The important implications of the residence time distributions and degree of mixing in MBR reactor on biodegradation of urban wastewater is analysed by a computer model. Adopted are the parameters (Table 2) and kinetic rate expressions from Activated Sludge Model No. 1 (ASM1).<sup>16–17</sup> Endogeneous oxidation of easily biodegradable waste by heterothropic active sludge is only considered. Applied is Monod kinetic model with bisubstrate limitation on waste substrate and dissolved oxygen. The balance equations are given by eq. 6 – 8. Applied is NDSolve program for numerical integration given by software *Mathematica*.<sup>18</sup> The results of integration correspond to the assumptions of the ideally

Table 2 – Model parameters for biodegradable endogeneous oxidation of municipal wastewater by heterothropic activated sludge.<sup>16–17</sup>

Model parameters		
reactor volume	$V$	44 L
inlet concentration	$\gamma_{Su}$	400 mg $\text{O}_2 \text{ L}^{-1}$
Monod saturation constant	$K_S$	20 mg $\text{O}_2 \text{ L}^{-1}$
yield coefficient	$Y_{X/S}$	0,67 mg $\text{O}_2 / \text{mg O}_2$
maximum specific growth rate	$\mu_m$	0,25 $\text{h}^{-1}$
volumetric $\text{O}_2$ transfer coefficient	$k_{1a}$	120 $\text{h}^{-1}$
specific rate of biomass autolysis	$k_d$	0,026 $\text{h}^{-1}$
oxygen saturation concentration	$\gamma_{\text{O}_2}^*$	8 mg $\text{O}_2 \text{ L}^{-1}$

Table 3 – Concentrations of soluble biodegradable substrate in outlet at various liquid and gas flowrates determined by the following models of liquid flow: ideal mixing (CSTR), segregated flow, the CSTR network, and the maximum mixidness model

$q_L/L\ h^{-1}$	$q_G/m^3\ h^{-1}$	Segregated $\gamma_S/mg\ O_2\ L^{-1}$	CSTR $\gamma_S/mg\ O_2\ L^{-1}$	Network model $\gamma_S/mg\ O_2\ L^{-1}$	Maximum mixidness $\gamma_S/mg\ O_2\ L^{-1}$
7.2	1.7	0.73	2.63	2.47	4.34
11.25	3.4	1.23	4.41	5.61	8.68
16.51	3.4	1.59	7.15	9.61	12.1

mixed reactor CSTR. In Table 3. are given the values at steady states at different liquid and gas flowrates and constant inlet concentration of waste substrate  $\gamma_{s,f} = 400\ mg\ O_2\ L^{-1}$ . In order to study the effect of nonideal flow and degrees of mixidness on the conversion of substrate determined are the upper and lower boundaries corresponding to the zero (segregated flow) and the maximum mixidness degree. Steady state outlet concentrations in case of the segregated flow are determined by integration of eq. 9–10. The integrands are the products of concentration profiles in a batch reactor and HRTD functions for a given liquid and gas flow. Results of integration are given in Table 3. and the obtained outlet concentrations are about 50 % lower compared to the previous case of ideal mixing. They are the concentration lower bounds, i.e. the highest values of substrate conversions attainable at given HRTD function. Assumption of the maximum mixidness provides the upper bound for outlet concentrations. The balances eq. 13 – 14 involve the life expectancy functions  $\Lambda(t)$  defined by eq. 2. In Fig. 10 are presented typical examples for several combinations of liquid and gas flows. Their asymptotic values correspond to the reciprocal values of the average residence time. The balance equations 13 – 14 have specified the right hand boundary con-

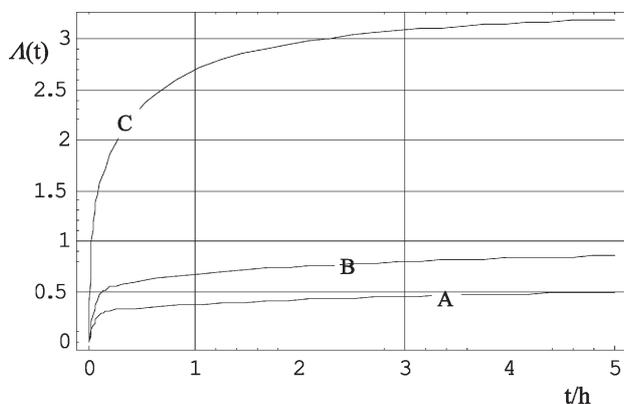


Fig. 10 – Life expectancy functions for the experiments at the following liquid and gas flow rates: A)  $q_L = 7.2\ L/h$ ,  $q_G = 5.1\ m^3/h$ ; B)  $q_L = 11.25\ L/h$ ,  $q_G = 3.4\ m^3/h$ , C)  $q_L = 36\ L/h$ ,  $q_G = 5.1\ m^3/h$

dition at asymptotic condition  $\lambda = \infty$ , eq. 15 – 16. The steady state concentrations are found at the opposite boundary  $\lambda = 0$ , eq. 17. The asymptotic missing values of concentrations  $\gamma(\lambda = \infty)$  have to be supplied by evaluation of the nonlinear algebraic equations 18–19. Results are given in Table 3. The obtained concentrations are the maximum values for given HRTD functions.

Besides the upper and lower bounds, and the ideal mixing case, also is applied the network model developed by the least square procedure. The balances are simultaneously integrated for the three interconnected CSTR reactors (Fig. 4) until a steady state are achieved and are given in Table 3.

Comparison of the results confirms the lower and upper boundaries determined by the models of zero mixing (segregated flow) and maximum mixidness, hence the results by the ideal mixing and the network model fall into the range defined by the boundaries. The values predicted by the network model are approximately in the middle of the determined bounds for each case of gas and liquid flowrate.

## Conclusions

Probability density functions of liquid residence time in two phase membrane square shaped laboratory scale reactor for wastewater treatment are experimentally determined by tracer experiments.

Obtained functions are successfully modelled by a simple CSTR network of two parallel zones with slow and fast flow, followed by a joint mixing zone. The average error between model predictions and experimental HRTD values are about 5 %.

The effect of residence time distribution and degree of mixing on biodegradation of urban wastewater is analysed by a computer model. The models of zero mixing (segregated flow) and maximum mixidness determine the boundaries of substrate conversion. Obtained are significant differences in the steady state conversions depending on the de-

gree of mixing. Importantly, the most commonly used model of ideal mixing predicts significantly higher substrate conversions compared to the expected values estimated by the network model.

The obtained results are applicable in development and design of large scale membrane reactor for municipal wastewater processing.

### Nomenclature

$b$	– model parameter
$E$	– density of residence time distribution
$h$	– height, m
$HRT$	– density of hydraulic residence time distribution
$k_d$	– specific rate of biomass autolysis, $\text{h}^{-1}$
$k_a$	– volumetric oxygen transfer rate, $\text{h}^{-1}$
$K$	– Monod's saturation constant, $\text{mg O}_2 \text{ L}^{-1}$
$q$	– flow rate, $\text{m}^3 \text{ h}^{-1}$
$R$	– correlation coefficient
$s$	– Laplace variable
$t$	– time, h
$v$	– spatial velocity, $\text{m s}^{-1}$
$W$	– transfer function
$Y$	– yield factor, $\text{mg O}_2 / \text{mg O}_2$
$\alpha$	– significance level
$\varepsilon$	– flow distribution coefficient
$\gamma$	– mass concentration, $\text{mg O}_2 \text{ L}^{-1}$
$\Lambda$	– density of life expectancy distribution
$\lambda$	– time, h
$\mu$	– biomass specific growth rate, $\text{h}^{-1}$
$\tau$	– average residence time, h

### Subscripts

$B$	– batch
$f$	– feed
$G$	– gas
$in$	– inlet
$L$	– liquid

$m$	– maximum
$s$	– biodegradable substrate
$o$	– oxygen
$out$	– outlet
$x$	– biomass

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